Preface

Nowadays, “Power Electronics,” basically deals with conversion and control of electrical power using electronic converters based on semiconductor power switches. Historically, the evolution of power electronics has generally followed the semiconductor power device evolution. Power solid-state devices are the heart and soul of modern power electronics equipment. Therefore, the age of power solid-state electronics is often called the second electronics revolution. Development of microelectronic controllers has made revolutionary advances in power electronics.

Power electronics circuits are an integral part of all electronics equipments. Power supply is the heart of all electronic circuits. For low-power consumption units or for portable operation, a battery is often used. For example, in a power supply system for a laptop computer, DC/DC converter converts lithium battery voltage into the output voltages required by the load. AC mains supply is generally used as a primary power supply for high power circuits. In almost all cases, this power requires conversion to the appropriate DC voltage by AC to DC converters. Besides DC to DC and AC to DC converters, typical applications of power electronics include conversion of an unregulated DC voltage to a regulated one, conversion of DC to AC, and conversion of an AC power source from one amplitude and/or frequency to another amplitude and/or frequency.

DC to DC converters and DC to AC inverters provide natural interfaces with direct energy sources such as solar cells, thermoelectric generators, fuel cell uninterruptible power sources. Commercial applications of power electronics include industrial motor drives, electrical vehicle power and drive system, as communications equipment, off-line power systems for computers, robotic technology, inverter systems for renewable energy generation applications, etc. In the twenty-first century, power electronics will have a large impact on industrial automation, energy conservation, utility systems, transportation, and environmental protection.

Power electronics includes application from ranges less than one watt (battery-operated portable equipment) to more than a few 100 or 1,000 W in motor drives or in rectifiers and inverters that interface DC transmission lines to the AC utility power system. In view of the fact that high efficiency is essential in all power processing applications, the key element is the switching converter. A small power

loss and hence high energy efficiency cannot be met by linear electronics where the semiconductor devices are operated in their active (linear) region. That is the reason that switched mode of semiconductor devices (transistors or thyristors) are used in switching converters. When a switch operates in the off state, its current is close to zero, and when it operates in the on state, its voltage drop is very small. In either state, its power dissipation is low. If the switching device is ideal, either the device voltage in on state or the device current in off state is zero so that power dissipation is also zero. Efficiency depends on switching frequency because real devices absorb some power when transition between on and off states and vice versa. Efficiency is improved by use of new switching devices, new circuit topologies, modern control techniques, and new ways of manufacture.

The book "Power Electronics: Converters and Regulators" is structured into ten chapters.

Chapter 1 is "Introduction," and briefly reviews parts of signals and systems theory as used in power electronics, as well as some circuit theory and basic components used in power electronics.

Chapter 2 covers "Diodes and Transistors," and particularly covers their use as switches in power electronics circuits. Power MOS transistors, IGBT and some standard driver and snubber circuits are also described in this chapter.

Chapter 3 is still focused on devices, "Regenerative Switches." The most important regenerative switches are covered including new powerful devices such as the Emitter Turn-Off Thyristor (ETO) and Insulated Gate Bipolar Thyristor (IGCT).

Coverage of topics specific to power electronics starts with Chap. 4, "PWM DC/DC Converters." All basic topologies are analyzed in both Continuous (CCM) and Discontinuous Current Mode (DCM). This chapter also includes discussion of loss mechanisms in these converters.

"Control Modules" are presented in Chap. 5. Basic principles and characteristics of PWM control modules are covered. The chapter describes a number of circuits used to control power electronic systems, and illustrates their application.

Chapter 6 covers "DC/AC Converters," i.e., inverters. One-phase and three-phase bridge inverters are presented. Also, the most used control techniques are discussed, unipolar and bipolar PWM and space vector modulation.

Chapter 7 is followed by its natural complement, AC/DC Converters, i.e., rectifiers. The coverage starts from uncontrolled rectifiers, progressing toward phase controlled rectifiers and high power factor PWM rectifiers. The most commonly used control techniques are presented, as well as some application with the PWM rectifiers.

Chapter 8 covers "AC/AC Converters." This chapter describes single-phase and three-phase AC/AC voltage converters and both direct and indirect frequency converters. Also, an overview of matrix converters and their applications is presented in this chapter.

Chapter 9 contains description of "Resonant Converters." Many topologies are covered: series resonant converter, parallel resonant converter, class E resonant

converters, zero voltage and zero current switching converters, and some control circuits used in resonant converters.

Chapter 10 covers "Multilevel Converters." Basic topologies of DC/DC and DC/AC multilevel converters are presented. Also, some widely used control techniques, such as multilevel PWM, space vector modulation and selective harmonic elimination, are briefly discussed in this chapter.

The book "Power Electronics: Converters and Regulators" is primarily intended for students of electrical engineering. A significant part of the book was created from authors' teaching materials for the subjects *Pulse Electronics* and *Power Electronics* at the Faculty of Electrical Engineering, University of Banja Luka in the last 15 years. This is third revised and updated edition. In relation to the two previous issues from 2000 and 2007, which were intended for Serbian and Croatian language readers (ex-Yugoslavia countries), this issue has have more than one third of the content altered. The alterations are in the form of: completely or partially new chapters, such as Multilevel Converters, Space Vector Modulation, Active Rectifier, PWM Rectifiers, Matrix Converters, Power Factor Correction, and number of problems at the end of every chapter.

For the design of power electronic converters, different knowledge from electrical engineering fields is required, such as theory of electrical circuits, electronics, electromagnetics, theory of control systems, and heat transfer. In addition, semiconductor elements in switched mode are highly nonlinear, and analysis of the circuits is quite complex. Therefore, simplified models are used in this book with explanation of the basic processes and essential phenomena. This is followed by waveforms of characteristic voltages and currents, which should complete understanding of electrical circuits operation. Numerous solved examples in each chapter should help students better understand the book material. Besides, we used examples to introduce ways of thinking about the problems, methods of analysis, and use of approximations. For some problems the results obtained by PSPICE simulation are presented. At the end of each chapter, unsolved problems are given, which should help the students to test their knowledge and stimulate thinking about the material presented in the chapter.

The authors thank their colleagues Prof. Predrag Pejović from the Faculty of Electrical Engineering in Belgrade and Prof. Vladimir Katić from the Faculty of Technical Sciences in Novi Sad, whose suggestions significantly contributed to the content of this book. Also, we thank Dr. Vojislav Aranđelović from the Institute of Nuclear Physics Vinča—Belgrade and Dr. Zoran Jakšić from the Institute of Physics—Belgrade, who with content and linguistic corrections improved the intelligibility of the text as a whole.

Branko L. Dokić
Branko Blanuša

Contents

1 **Introduction** . . . 1
 1.1 Types of Signals . . . 1
 1.2 Root-Mean-Square and Average Values of Periodic Signals . . . 5
 1.3 Power of Periodic Currents . . . 9
 1.4 Switching Elements . . . 16
 1.5 Magnetic Elements . . . 24
  1.5.1 Chokes . . . 26
  1.5.2 Transformers . . . 27
 1.6 Capacitors . . . 30
 1.7 Radio-Frequency Interference . . . 33
 1.8 Cooling of Components . . . 35
 References . . . 41

2 **Diodes and Transistors** . . . 43
 2.1 Diode as a Switch . . . 43
  2.1.1 The Temperature Characteristics . . . 47
  2.1.2 Dynamic Diode Characteristics . . . 50
  2.1.3 Schottky Diodes . . . 54
  2.1.4 The Selection of Pulse Diodes . . . 56
 2.2 Bipolar Transistor as a Switch . . . 58
  2.2.1 The Cut Off Region . . . 59
  2.2.2 The Saturation Region . . . 67
  2.2.3 Static Transfer Characteristic . . . 72
  2.2.4 Dynamic Inverter Characteristics . . . 75
  2.2.5 Nonsaturated Switch . . . 92
  2.2.6 Capacitatively Loaded Inverter . . . 96
  2.2.7 Inductively Loaded Switch . . . 101
  2.2.8 Transistor Selection . . . 110
  2.2.9 Driver Circuits . . . 116

2.3 Power MOS Transistor as Switch 117
2.3.1 Power VDMOS Transistor 119
2.3.2 Power BiMOS Switch 121
2.3.3 Static Parameters 123
2.3.4 Safe Operation Area 138

3 Regenerative Switches 143
3.1 Unijunction Transistor 145
3.1.1 Temperature Characteristics 148
3.1.2 Programmable Unijunction Transistor 150
3.1.3 Complimentary UniJunction Transistor 154
3.1.4 Pulse Generators 156
3.1.5 Non-standard Applications 161
3.2 Thyristors 166
3.2.1 Triode Thyristor—SCR 166
3.2.2 Gate Assisted Turn-Off Thyristor 188
3.2.3 Asymmetric Thyristor 189
3.2.4 Reverse Conducting Thyristor 189
3.2.5 Gate Turn-Off Thyristor. 190
3.2.6 MOS Thyristor 191
3.2.7 Insulated Gate Control Thyristor 192
3.2.8 Emitter Turn-Off Thyristor 196
3.2.9 Photo-thyristor 199
3.2.10 Unilateral Switch 199
3.2.11 Double Switch—SBS 200
3.2.12 Diode Thyristors 200
3.2.13 TRIAC 203
References. 209

4 PWM DC/DC Converters 211
4.1 Forward Converters. 213
4.1.1 Analysis of the Basic Circuit 214
4.2 Galvanically Isolated Forward Converter 240
4.3 Boost Converter 246
4.3.1 Analysis of the Basic Scheme. 246
4.3.2 Variation of the Output Voltage 252
4.3.3 Boundary Between the Continuous and the Discontinuous Mode 255
4.3.4 Discontinuous Mode 256
4.3.5 Power Losses 258

4.4 Indirect Converter 260
4.4.1 Boundary Between the Continuous and the Discontinuous Mode 263
4.4.2 Discontinuous Mode 263
4.4.3 Indirect Converter with Galvanic Separation 267
4.5 Push–Pull (Symmetric) Converters 275
4.5.1 Analysis of Idealized Circuit in Continuous Mode 277
4.5.2 Output Characteristics 285
4.5.3 Selection of Components 288
4.5.4 DC Premagnetization of the Core 296
4.5.5 Half-Bridge Converter 297
4.5.6 Bridge Converter 298
4.5.7 Hamilton Circuit 301
4.6 Ćuk Converters 302
4.6.1 Elimination of the Current Ripple 305
4.6.2 Ćuk Converters with Galvanic Isolation 306
References 309

5 Control Modules 311
5.1 Basic Principles and Characteristics of PWM Control Modules 312
5.1.1 Circuit Analysis 314
5.1.2 Simple PWM 317
5.2 Voltage-Controlled PWM 323
5.3 Current-Controlled PWM 324
5.3.1 Compensated PWM 327
5.4 IC Control Modules 330
5.4.1 Control Module TL494 337
5.4.2 Control Module SG1524/2524/3524 341
5.4.3 Control Module TDA 1060 352
Reference 357

6 DC/AC Converters–Inverters 359
6.1 Single-Phase Voltage Inverters 360
6.1.1 Pulse-Controlled Output Voltage 365
6.2 Pulse-Width Modulated Inverters 368
6.2.1 Unipolar PWM 373
6.3 Three-Phase Inverters 377
6.3.1 Overmodulation ($m_a > 1$) 383
6.3.2 Asynchronous PWM 384

6.4 Space Vector Modulation . . . 384
6.4.1 Space Vector Modulation—Basic Principles . . . 384
6.4.2 Application of Space Vector Modulation Technique . . . 387
6.4.3 Direct and Inverse Sequencing . . . 390
6.5 Real Drive Influence . . . 391
References . . . 394

7 AC/DC Converters–Rectifiers . . . 395
7.1 Half-Wave Single-Phase Rectifiers . . . 396
7.2 Full-Wave Rectifiers . . . 397
7.2.1 Commutation of Current . . . 400
7.3 Output Filters . . . 404
7.3.1 Capacitive Filter . . . 404
7.3.2 L Filter . . . 408
7.4 Voltage Doublers . . . 410
7.5 Three-Phase Rectifiers . . . 411
7.6 Phase Controlled Rectifiers . . . 416
7.6.1 Full-Wave Thyristor Rectifiers . . . 417
7.6.2 Three-Phase Thyristor Bridge Rectifiers . . . 424
7.7 Twelve-Pulse Rectifiers . . . 426
7.8 Rectifiers with Circuit for Power Factor Correction . . . 429
7.9 Active Rectifier . . . 432
7.9.1 Active Rectifier with Hysteresis Current Controller . . . 433
7.10 PWM Rectifiers . . . 436
7.10.1 Advanced Control Techniques of PWM Rectifiers . . . 441
7.10.2 PWM Rectifier with Current Output . . . 445
7.10.3 PWM Rectifiers in Active Filters . . . 450
7.10.4 Some Topologies of PWM Rectifiers . . . 450
7.10.5 Applications of PWM Rectifiers . . . 452
References . . . 455

8 AC/AC Converters . . . 457
8.1 Single-Phase AC/AC Voltage Converters . . . 457
8.1.1 Time Proportional Control . . . 464
8.2 Three-Phase Converters . . . 466
8.3 Frequency Converters . . . 468
8.3.1 Direct Frequency Converters . . . 468
8.4 Introduction to AC/AC Matrix Converters . . . 478
8.4.1 Basic Characteristics . . . 478
8.4.2 Bidirectional Switches . . . 481

8.4.3 Realization of Input Filter 482
8.4.4 Current Commutation 483
8.4.5 Protection of Matrix Converter 486
8.4.6 Application of Matrix Converter 488
References. 491

9 Resonant Converters 493
9.1 Resonant Circuits 495
9.2 Resonant Converters of Class D 499
9.2.1 Series Resonant Converters 501
9.2.2 Parallel Resonant Converters 509
9.2.3 Series–Parallel Resonant Converter 512
9.3 Series Resonant Converters Based on GTO Thyristors 514
9.4 Class E Resonant Converters 518
9.5 DC/DC Converters Based on Resonant Switches 521
9.5.1 ZCS Quasi-resonant Converters 523
9.5.2 ZVS Quasi-resonant Converters 531
9.5.3 Multiresonant Converters 537
9.6 ZVS Resonant DC/AC Converters 539
9.7 Soft Switching PWM DC/DC Converters. 540
9.7.1 Phase Shift Bridge Converters 541
9.7.2 Resonant Transitions PWM Converters 547
9.8 Control Circuits of Resonant Converters 551
9.8.1 Integrated Circuit Family UCx861-8 553
9.8.2 Integrated Circuits for Control of Soft Switching PWM Converters 556

10 Introduction to Multilevel Converters 559
10.1 Basic Characteristics 559
10.2 Multilevel DC/DC Converters. 563
10.2.1 Time Interval: $nT < t < nT + DT$, $n = 0, 1, 2,\ldots$ 565
10.2.2 Time Interval: $nT + DT < t < (n + 1)T$ 565
10.3 Multilevel Inverters 573
10.3.1 Cascaded H-Bridge Inverters 573
10.3.2 Diode-Clamped Multilevel Inverters 578
10.3.3 Flying Capacitor Multilevel Inverter 580
10.3.4 Other Multilevel Inverter Topologies 582
10.4 Control of Multilevel Inverters 585
10.4.1 Multilevel SPWM 586

10.4.2 Space Vector Modulation 589
10.4.3 Space Vector Control 590
10.4.4 Selective Harmonic Elimination 591
References 592

Bibliography 593

Index 595

Chapter 2
Diodes and Transistors

In all basic circuits of the pulse DC/DC or DC/AC voltage converters, the switching elements are transistors (bipolar and unipolar) and diodes. In the analysis of a basic circuit, transistors and diodes have been considered as ideal switches (zero on-resistance, infinite off-resistance, and instantaneous transition from one state to the other). However, they are not ideal switches but have real parameters, in both the static and dynamic modes of operation. The influence of these parameters on the characteristics of pulse converters is considerable, particularly on the efficiency factor. For this reason, in this chapter, a description is given of the basic switching characteristics of transistors (bipolar and unipolar) and diodes. An analysis is presented of the modes of control of transistor switches and the optimum control circuits are given. The analysis is of a general character and applies to all pulse assemblies using diodes or transistors as switches.

2.1 Diode as a Switch

The static characteristic of a p-n junction diode is nonlinear and is determined by

$$I_d = I_s\left(e^{V_d/m_d\varphi_t} - 1\right) \tag{2.1}$$

where I_s is the reverse saturation current, m_d is the correction factor ($m_d = 2$ for small currents—in the vicinity of the knee of the characteristic and $m_d = 1$ at higher currents), ϕ_t is the temperature potential. The static characteristic (Fig. 2.1) consists of three regions: conduction region (low-resistance), cut off (high-resistance), and breakdown. The region where the operating point is found depends on the voltage applied to the diode. Therefore, a diode can be used as a switch because its resistance can be controlled by the applied voltage.

When a diode is forward biased and if $V_d > V_{\mathrm{Dt}}$, where V_{Dt} is the conduction threshold voltage, the diode is on (conducting). Then its resistance is small (from 10 to 100 Ω). Since the threshold voltage of Si diodes is $V_{\mathrm{Dt}} = (0.5–0.6)$ V, in the conduction region $V_d \gg m\phi_t$, and $\exp(V_d/m_d\phi_t) \gg 1$, so the current is

© Springer International Publishing Switzerland 2015

B.L. Dokić and B. Blanuša, *Power Electronics*,
DOI 10.1007/978-3-319-09402-1_2

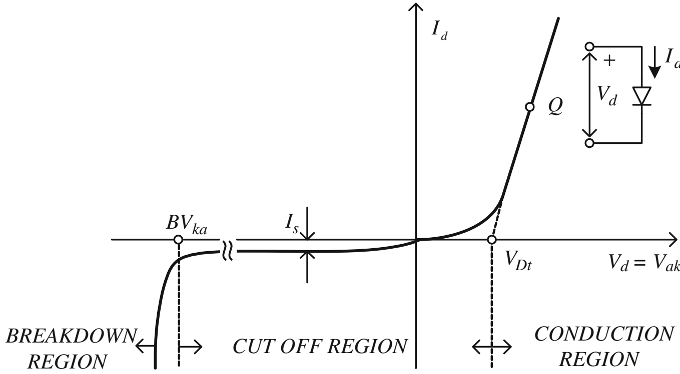

Fig. 2.1 The static diode characteristic

$$I_d \approx I_s\, e^{V_d/(m_d\,\varphi_t)} \tag{2.2}$$

The dynamic diode resistance is the reciprocal value of the dynamic diode conductance and is defined by

$$r_d = \left.\frac{1}{\mathrm{d}I_d/\mathrm{d}V_d}\right|_{V_d=\mathrm{const}} = \frac{m_d\,\varphi_t}{I_{\mathrm{DQ}}}, \tag{2.3}$$

where I_{DQ} is the diode current at the quiescent operating point Q. Any increase of the diode current I_{DQ} decreases the dynamic resistance. For instance, for $I_{\mathrm{DQ}} = 1$ mA, $r_d = 26\ \Omega$ and for $I_{\mathrm{DQ}} = 26$ mA, $r_d = 1\ \Omega$. It has been assumed that $\phi_t = 26$ mV and $m_d = 1$. It should be emphasized that (2.3) is the p-n junction resistance. The total resistance between the anode and the cathode is increased by the resistance of the base (substrate), which is typically 10–100 Ω, i.e., $R_d = r_d + r_b$. At high currents the resistance r_b is dominant and the *V-I* characteristic in that region is almost linear.

In many practical applications a conducting diode can be approximated, with a satisfactory accuracy, by a straight line of the slope determined by R_{D} and a voltage source V_{Dt} (Fig. 2.2a). Then

$$V_d = V_{Dt} + R_D I_D. \tag{2.4}$$

On the other hand, in the majority of diode applications as a switch, the resistance of the driving circuit, which determines the current I_{DQ} in the quiescent operating point Q, is much higher than R_{D} so that the voltage variation across the diode is negligible. The diode is then replaced by a voltage source V_{D} and its characteristic is drawn as a straight line that passes through the operating point Q and is orthogonal to the V_{D} axis (Fig. 2.2b). Typically $V_{\mathrm{D}} = 0.7$–0.8 V and includes a voltage drop of 0.1–0.2 V across R_{D}.

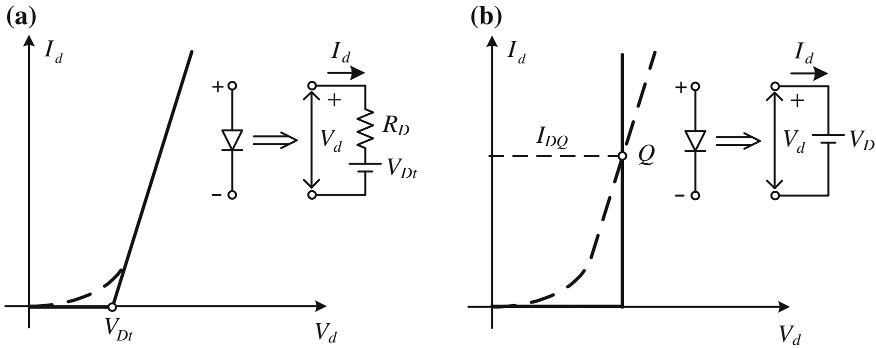

Fig. 2.2 The practical approximations of the V-I characteristic in the conduction region and the corresponding diode equivalent circuits (*dashed line*—real characteristic)

When a diode is reverse biased, i.e., $V_{AK} < 0$, and if $|V_{AK}| > m_d\varphi_t$, then $\exp(V_D/m_d\varphi_t) \ll 1$ and the current through the diode is equal to the reverse saturation current $I_{DF} = -I_S$. Namely, already at $V_{AK} = -0.2$ V from (2.1) it follows that $I_D = -0.98 I_S$. This means that at very small reverse voltages the cathode-anode current is saturated at $-I_S$. The measurements, however, indicate that the reverse current is considerably larger than I_S. This difference is largely due to generation-recombination of charge carriers in the transition region of the p-n junction. At reverse bias, the concentration of charge carriers in the depleted region drops well below the equilibrium concentration. Consequently, recombination is decreased and generation prevails. Owing to the generation of electron–hole pairs a reverse current proportional to the volume of the depleted region Sd and the rate of generation of pairs $G = n_i/(2\tau_o)$ arises, i.e.,

$$I_G = Sq\frac{n_i}{2\,\tau_0}d, \tag{2.5}$$

where S is the p-n junction area, d is the width of the transition region, n_i is the intrinsic concentration of free charge carriers, τ_0 is the lifetime of carriers in the transition region. It is well known that the width of the transition region increases with increasing the reverse bias, thus causing the increase of the reverse current due to increased generation of the electron–hole pairs:

$$I_G = Sq\frac{n_i}{2\,\tau_0}d_0\left(1 - \frac{V_I}{\varphi_k}\right)^n, \tag{2.6}$$

where V_I is the reverse voltage, φ_k is the contact potential, d_0 is the width of the depletion region at $V_I = 0$, $n = 1/2$ for abrupt and $n = 1/3$ for a p-n junction with a linear distribution of impurities.

The reverse saturation current I_S obtained on the basis of the diffusion theory of the p-n junction is determined by

$$I_s = Sq\left(\frac{D_p p_{n0}}{L_p} + \frac{D_n n_{p0}}{L_n}\right), \tag{2.7}$$

where D_p, L_p, and D_n, L_n are the respective diffusion coefficients and the diffusion lengths for the holes and electrons, respectively, p_{no} is the concentration of holes in n-type semiconductor, n_{po} is the concentration of electrons in p-type semiconductor. In the majority of practical applications the p-n junction is highly asymmetric since the concentration of holes p_{po} in the p-type region is much higher than the concentration of electrons in the n-type region. Therefore, a p^+n^- junction is the most frequent one. Then, $p_{po} \gg n_{no}$ and the hole current in (2.7) is much higher (several orders of magnitude) than the electron current. The reverse saturation current is thus

$$I_s \approx I_{sp} = \mathrm{Sq}\, D_p \frac{p_{n0}}{L_p}. \tag{2.8}$$

Applying the relations $p_{p0}n_{n0} = n_i^2$ and $L_p = D_p\tau_0$ one obtains:

$$\frac{I_G}{I_s} = \frac{1}{2}\frac{d}{L_p}\frac{n_{n_0}}{n_i}. \tag{2.9}$$

For instance, for a silicon diode with the following parameters: $d = 10^{-4}$ cm, $L_p = 2 \times 10^{-2}$ cm, $n_{n0} = 2.5 \times 10^{15}$ cm^{-3}, and $n_i = 1.9 \times 10^{10}$ cm^{-3} the ratio of the reverse generation current I_G to the reverse saturation current I_S is $I_G/I_S \approx 300$. At voltages of approximately 10 volts this ratio may become several thousands. Therefore, the total reverse current of a diode is shown in Fig. 2.3.

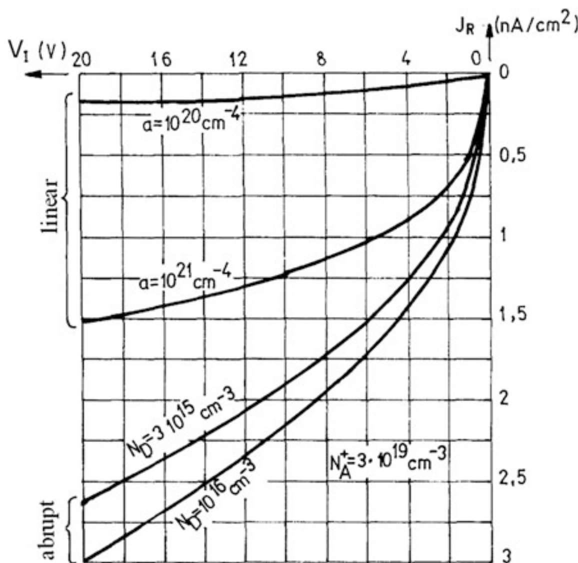

Fig. 2.3 Reverse current density versus reverse voltage for linear and abrupt p-n junction

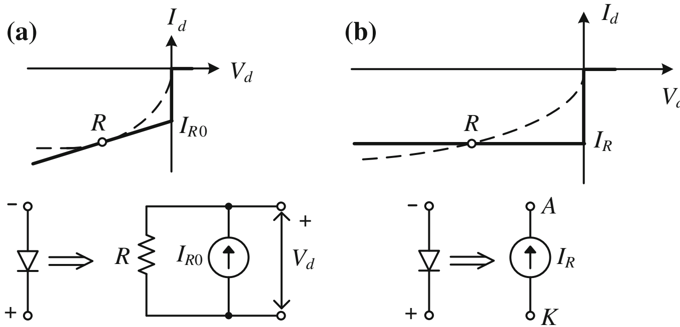

Fig. 2.4 The approximations of V-I characteristic and the corresponding equivalent circuits in the reverse region (*dashed lines*—real characteristics)

$$I_{\mathrm{I}} = I_{\mathrm{s}} + I_{\mathrm{G}} \approx I_{\mathrm{G}}\,. \tag{2.10}$$

Thus for linear and abrupt junctions the current density characteristic of a diode according to (2.6) is parabolic. In practice, however, the reverse characteristic of a diode is often replaced by a straight line tangential to the operating point R with a segment $-I_{R0}$ on the ordinate (Fig. 2.4a). This means that a diode can be replaced by a parallel connection of a current source I_{R0} and a leakage (reverse) resistance R_I (Fig. 2.4a). Then

$$I_d = -(I_{\mathrm{RO}} + V_d/R_I). \tag{2.11}$$

Reverse resistance R_I ranges from several tens kΩ (power diodes) to several hundreds MΩ. The resistance of the driving circuit is usually much lower than R_I, and the change of the reverse current can be neglected. The diode is then replaced by a current source $I_{\mathrm{RR}} > I_{R0}$ (Fig. 2.4b) which is specified for a given reverse voltage V_I. The reverse current I_{RR} is usually between 10^{-12} and 10^{-6} A for silicon diodes. Since this current is directly proportional to the surface of the p-n junction, this means that I_{RR} of power diodes is large and can be of the order of mA.

When the reverse voltage is higher than the breakdown voltage of a p-n junction, the diode behaves like a Zener diode if the current is limited. Typical values of the breakdown voltage are between several V up to 100 V. For a high voltage diode this voltage ranges from several 100 V up to several kV.

2.1.1 The Temperature Characteristics

The basic static parameters of a diode as a switch are the reverse current I_{R} when diode is not conducting and the forward bias voltage V_{D} when it is conducting. In many applications the temperature sensitivities of these parameters are of

considerable influence on the temperature sensitivities of the functional parameters of the circuits incorporating diodes.

The reverse current is approximated by (2.5). All parameters except n_i are constants independent on temperature, whereas the temperature dependence of n_i^2 is determined by:

$$n_i^2 = AT^3 e^{-V_g/\varphi_t}, \tag{2.12}$$

where T is temperature in K, $A = 1.5 \times 10^{32}\ \mathrm{cm^6\,K^3}$, V_g is the bandgap voltage and at room temperature it is 1.11 V. Voltage V_g is also temperature dependent and for silicon it is approximately determined by[1]

$$V_g(t) = V_{g0} - 3.6 \times 10^{-4}\ \mathrm{T}, \tag{2.13}$$

where $V_{g0} = 1.21$ V is the bandgap voltage for silicon at absolute zero. The reverse current thus can be written in the form

$$I_R = B\,T^{3/2}\,\mathrm{e}^{-V_g/(2\varphi_t)}, \tag{2.14}$$

where:

$$B = qSA^{1/2}\frac{d}{2\tau_0}. \tag{2.15}$$

After differentiating (2.14) over temperature and rearranging it, the temperature coefficient of the reverse current is obtained as

$$\frac{\mathrm{d}I_R}{I_R\mathrm{d}T} = \frac{1}{2T}\left(3 + \frac{V_{g0}}{\varphi_t}\right). \tag{2.16}$$

At room temperature T_0 = 300 K, the temperature potential is φ = 26 mV, V_{go} = 1.21 V and

$$\left.\frac{\mathrm{d}I_R}{I_R\mathrm{d}T}\right|_{T_0} = 0.0825\left[\frac{1}{^\circ\mathrm{C}}\right] \tag{2.17}$$

Very often, however, a more practical expression for $I_R = f(T)$ is used

$$I_R(T) = I_R(T_0)2^{\frac{T-T_0}{T_x}}, \tag{2.18}$$

where $I_R(T_0)$ is the current I_R at temperature T_0 and T_x is the temperature variation with respect to T_0, which doubles the value of I_R. T_x can be calculated by equating (2.14) and (2.18) which gives

$$\frac{3}{2}\ln\frac{T}{T_0}+\frac{V_g}{2\varphi_t}\left(\frac{\varphi_t}{\varphi_t(T_0)}-1\right)=\frac{T-T_0}{T_x}\ln 2. \tag{2.19}$$

If $(T - T_0)/T_0 \ll 1$ the logarithm of the temperature ratio can be written in the form

$$\ln\frac{T}{T_0}=In\left(1+\frac{T-T_0}{T_x}\right)\approx\frac{T-T_0}{T_x}. \tag{2.20}$$

Since $\varphi_t/\varphi_t(T_0) = T/T_0$, from (2.19) and (2.20) it follows that:

$$T_x=\frac{2\ln 2}{3+V_g/\varphi_t}T_0. \tag{2.21}$$

At room temperature (T = 300 K) for a silicon diode T_x = 9 °C. This would mean that the reverse current doubles for each 9 °C. Here the influence of the reverse saturation current is neglected. It can be shown that its variation with temperature is higher by a factor of 2 (it doubles for each 4.5 °C) since the generation current is proportional to n_i ($I_G \approx I_R \sim n_i$) whereas the injection current is proportional to n_i^2 ($I_S \sim n_i^2$). Due to the influence of the temperature variations of I_S it is accepted in practice that the total reverse current doubles for each 10 °C, i.e.

$$I_R=I_R(T_0)2^{\frac{T-T_0}{10\,^\circ\mathrm{C}}}. \tag{2.22}$$

It should be stressed that for small silicon diodes the current $I_R(T_0)$ is quite small and in many applications its temperature variation is of no importance.

The forward bias voltage V_d is also a function of temperature. The case of high currents I_D, when m_d = 1, will be considered first. Now the diffusion current compared to the generation-recombination current is dominant and using (2.2), (2.7), and (2.12), taking that $p_{n0}=n_i^2/N_D$ and $n_{p0}=n_i^2/N_A$, one obtains

$$V_d=V_g-\varphi_t\ln\frac{DT^3}{I_D}. \tag{2.23}$$

where D is a temperature independent constant. By differentiating V_d in terms of temperature at a constant current I_D it follows

$$\frac{\mathrm{d}V_d}{\mathrm{d}T}=\frac{V_d}{T}+\frac{\mathrm{d}V_g}{\mathrm{d}T}-\frac{\varphi_t}{T}\left(3+\frac{V_g}{\varphi_t}\right). \tag{2.24}$$

At room temperature for V_d = 0.7 V

$$\frac{\mathrm{d}V_d}{\mathrm{d}T} = \frac{700}{300} - 0.36 - \frac{26}{300}\left(3 + \frac{1110}{26}\right) = -2\frac{\mathrm{mV}}{°\mathrm{C}}.$$

Therefore, the temperature coefficient of the forward bias voltage is negative. Typically, V_d decreases by 2 mV if the temperature increases by 1 °C. It should be stressed that the expression (2.24) is general because it is also valid for low currents. Then $m_d = 2$ and the generation-recombination current proportional to n_i is dominant instead of I_S, so that

$$V_d = V_g - 2\varphi_t \ln\frac{PT^{3/2}}{I_D}, \tag{2.25}$$

where P is a temperature independent constant. By differentiating (2.25) in terms of T one obtains after rearrangement that at low currents $\mathrm{d}V_d/\mathrm{d}T$ is determined by (2.24). From (2.24), it is noticeable that the temperature coefficient depends on the position of the operating point. For instance, for $V_D = 0.6$ V, $\mathrm{d}V_d/\mathrm{d}T = 2.32$ mV/°C and for $V_D = 0.8$ V, $\mathrm{d}V_d/\mathrm{d}T = 1.66$ mV/°C. In general, it can be said that depending upon the operating regime the temperature coefficient of the forward bias voltage is within the range −1.5 to −2.5 mV/°C.

2.1.2 Dynamic Diode Characteristics

In addition to the static parameters in the switching regime, it is important to know the dynamic response of the diode. The dynamic characteristics of the diode are determined by the transition processes, i.e., the turn-on/turn-off transition times in response to a pulse drive.

Let the diode be driven through a resistance R by a pulse voltage source varying between $-V_2$ and V_I (Fig. 2.5). The turn-on processes at the instant $t = 0$ will be considered first. For $t < 0$ $V_I = -V_2$ the diode is reverse biased and off. A change of the input voltage at $t = 0$ turns on the diode. If R is much higher than the diode resistance, the current through the diode is

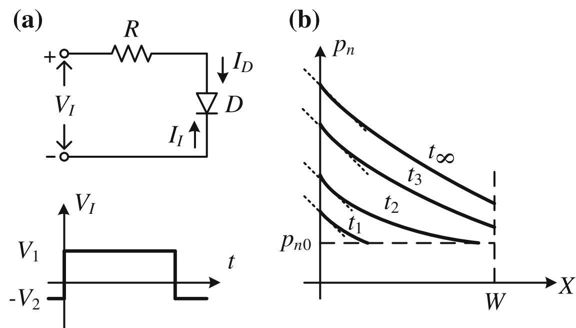

Fig. 2.5 The basic diode switching circuit (**a**) and distribution of holes in the *n*-region (**b**)

$$I_D = \frac{V_1 - V_D}{R} \approx \frac{V_1}{R}, \tag{2.26}$$

because $V_I \gg V_D$. It is known that the diode current is proportional to the gradient of the injected holes at the edge of the transition region, i.e.,

$$I_\mathrm{D} = -qSD_p \frac{\mathrm{d}p}{\mathrm{d}x}|_{x=0} = \frac{V_1}{R} = \text{const.} \tag{2.27}$$

During the turn-on process, the hole concentration at the edge of the transition region grows from the equilibrium concentration $p_{n\mathrm{o}}$ to the concentration determined by the steady state voltage applied across the diode. Since the current through the diode is constant, the change of the hole concentration at the edge of the transition region is also constant (Fig. 2.5b). Practically the steady state is established after a period somewhat longer than the lifetime of the holes, τ_p.

In general, the spatial concentration of holes in the n region during turn-on (Fig. 2.5b) is determined by the diffusion equation:

$$D_p \frac{\partial^2(\Delta p_n)}{\partial x^2} = \frac{\partial(\Delta p_n)}{\partial t} + \frac{\Delta p_n}{\tau_p}, \tag{2.28}$$

where $\Delta p_n = p_n - p_{\mathrm{n}0}$ is the excess hole concentration in the n region. The equation governing the excess hole charge can be obtained if Eq. (2.28) is multiplied by $qS\mathrm{d}x$ and integrated along the neutral region from $x = 0$ to $x = w$. Therefore:

$$qSD_p \left[\left. \frac{\partial(\Delta p_n)}{\partial x} \right|_w - \left. \frac{\partial(\Delta p_n)}{\partial x} \right|_0 \right] = \frac{\mathrm{d}}{\mathrm{d}t} \int_0^w qS\Delta p_n \, \mathrm{d}x + \frac{1}{\tau_p} \int_0^w eS\Delta p_n \, \mathrm{d}x. \tag{2.29}$$

Since the hole charge is determined by

$$Q_p = \int_0^w qS\Delta p_n \mathrm{d}x \tag{2.30}$$

and taking into account (2.27), Eq. (2.29) can be written in the form

$$I_\mathrm{p}(0) - I_\mathrm{p}(w) = \frac{\mathrm{d}Q_p}{\mathrm{d}t} + \frac{Q_\mathrm{p}}{\tau_\mathrm{p}}. \tag{2.31}$$

In the same way, it is possible to derive the equation for the excess electrons in the neutral region. In practice, however, the impurity concentrations of the two sides of the junction are distinctly asymmetric, thus $N_A \gg N_D$, $p_{\mathrm{n}0} \gg n_{\mathrm{p}0}$ and consequently $Q_p \gg Q_n$. This indicates that the influence of holes on the dynamic process is dominant. Due to this, the index "p" in (2.31) can be replaced by "d". Since $I_p(0) - I_p(w) \approx I_p(0) = I_d$ it follows

$$I_d = \frac{dQ_d}{dt} + \frac{Q_d}{\tau_d}. \quad (2.32)$$

This is the charge control equation and it represents the basic relation between the diode current and the excess minority carriers charge in the dynamic operating mode of the diode. In the steady state for a conducting diode $dQ_d/dt = 0$ and $I_d = Q_d/\tau_d$. The injection process is in equilibrium with the recombination process. In other words, a dynamic equilibrium between injection and recombination has been established. The amount of charge in the vicinity of the transition region is constant and proportional to the current through the diode.

At the instant t_o the input voltage abruptly changes from $+V_1$ to $-V_2$. The diode is reverse biased, but the current through the diode does not drop immediately to the value of the reverse current. On the contrary, for some time the diode conducts with low resistance in the reverse direction (cathode–anode), the current being determined by the external elements. Namely, if $V_d \ll V_2$, the reverse current through the diode is

$$I_1 \approx V_2/R_2 = \text{const.} \quad (2.33)$$

When, at the instant $t = t_o$ the diode is abruptly reverse biased, the direction of the field applied to the p-n junction will change. The direction of the applied field is from the cathode towards the anode and it supports movement of the minority charge carriers. Thanks to this, the excess holes from the *n* region return to the *p* region. The direction of the change of the hole concentration at the edge of the junction is altered (Fig. 2.6a). Since the current is constant, the slope of the hole concentration at the edge of the junction is also constant. During this time the piled up charge clears away. A constant reverse current (low diode resistance) will exist as long as the hole concentration at the edge of the junction is greater than zero.

The duration of this phenomenon is called the discharge time, often the accumulation or storage time. It is denoted by t_s. Therefore, during t_s the charge piled up in the vicinity of the p-n junction clears away, i.e., during t_s the diode retains low resistance.

Putting $I_d = I_I$ into Eq. (2.32) and using the initial condition $Q_d(0) = \tau_d I_D$, one obtains that the change of the excess hole charge is

$$Q_d(t) = \tau_d(I_D + I_I)e^{-t/\tau_d} - I_I\tau_d. \quad (2.34)$$

From the condition $Q_d(t_s) = 0$ and (2.34) the storage time is

$$t_s = \tau_d \ln(1 + I_D/I_I). \quad (2.35)$$

Therefore, the storage time is lower if the forward current is lower (Fig. 2.7a), since the stored charge is lower. On the other hand, the storage time is lower if the reverse current is higher (Fig. 2.7b), because the process of clearing away the stored charge is faster.

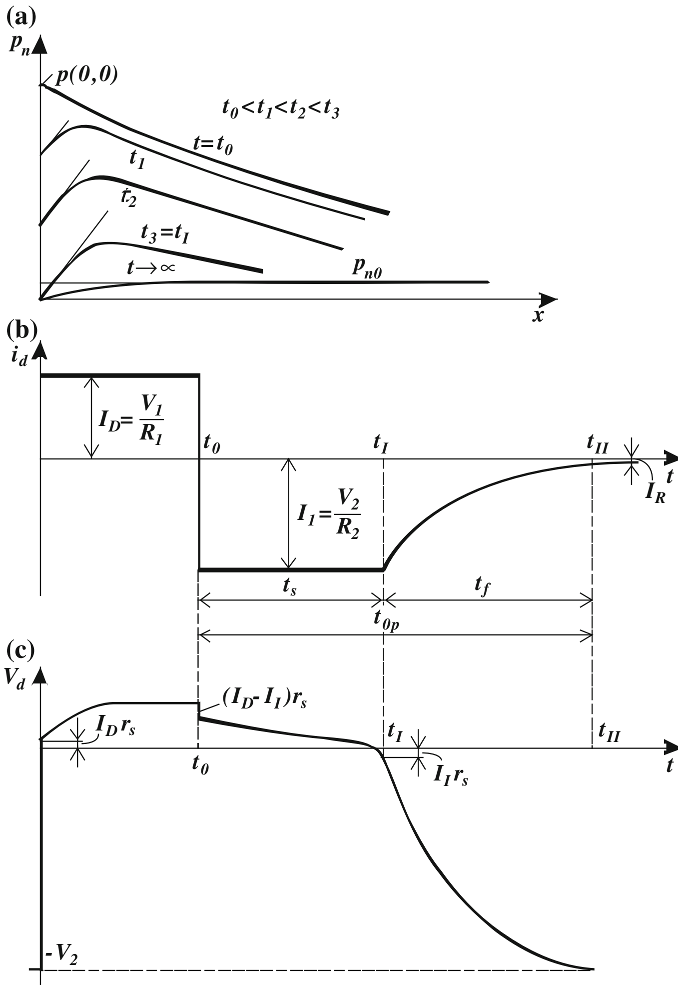

Fig. 2.6 Variation of the hole concentration (**a**), current (**b**), and voltage (**c**) during the turn-off process

Figure 2.6 shows the variations of the current and voltage of a diode in the transient regime. At the instant $t = t_{\rm o}$ voltage undergoes a negative swing

$$\Delta V = r_s(I_D - I_I). \tag{2.36}$$

Here r_s is the Ohmic resistance of the diode and $I_D - I_I$ is the current swing through the diode at the initial moment. During $t_{\rm s}$ the voltage across the diode drops to a value $-r_sI_I$. After t_s the diode is obviously reverse biased. The turn-off process continues until the reverse current I_R is attained. The current through the diode reduces and the voltage across it grows more negative. This time is called the fall time and is denoted

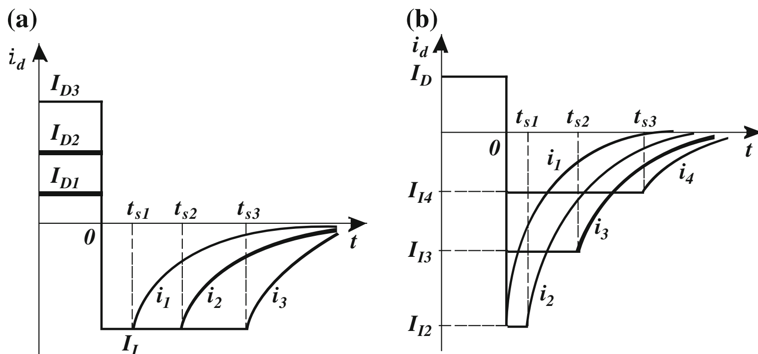

Fig. 2.7 The illustration of the dependence of storage time on forward current for a constant reverse current (**a**) and on reverse current for a constant forward current (**b**)

by t_f. Practically, during t_f the capacitor formed by the reverse biased p-n junction is charged. The total turn-off time of the diode $t_s + t_f$ is called the recovery time (denoted by t_r), or sometimes the reverse recovery time (denoted by t_{rr}).

It has been shown that the duration of the transient process is directly proportional to the lifetime of the minority carriers. For this reason in order to reduce τ_p in the *n* region in fast diodes one introduces the recombination centers, most frequently the atoms of gold. In this way, it is possible to obtain $\tau_p < 1$ ns. Fast diodes, however, have larger reverse saturation currents and lower breakdown voltages. The lifetime of holes in diodes with gold atoms increases with temperature, namely:

$$\tau_p(T) = \tau_p(T_0)(T/T_0)^r, \tag{2.37}$$

where *r* is a constant; for low injections in silicon it amounts to 3.5 and in germanium 2.2. For instance, for a temperature increase from 213 K (−60 °C) to 353 K (+80 °C) the average lifetime in silicon increases nearly six times. The duration of the transient process thus largely depends upon temperature.

2.1.3 Schottky Diodes

It is known that the junction of a metal and a weakly doped semiconductor possesses rectifying properties. For instance, at an aluminum—*n*-type silicon junction, when the donor concentration in silicon is $N_D < 5 \times 10^{18}$ cm^{-3}, a Schottky barrier is formed and the junction is conductive in one direction and nonconductive in the other. A structure metal—*n*-type semiconductor with typically $N_D \leq 10^{16}$ cm^{-3} makes a Schottky diode.

A Schottky barrier at a metal-semiconductor junction depends upon the type of metal and is within limits 0.58 and 0.85 V. This barrier, similarly to that of a p-n junction, prevents the diffusion of electrons from metal to semiconductor in a

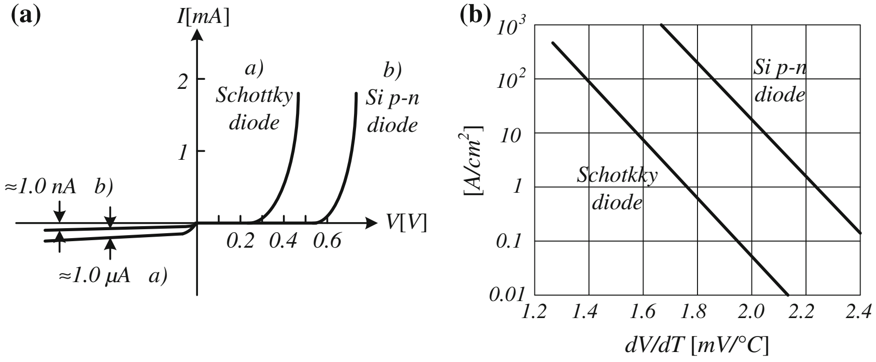

Fig. 2.8 V-I characteristic of Schottky diode (**a**) and temperature coefficient of forward voltage (**b**)

nonbiased diode. A positive polarization of the diode (metal is at a higher potential) decreases the potential barrier and electrons from semiconductor cross over to metal. A reverse polarization increases the potential barrier and widens the space charge region which prevents the movement of electrons. The diode is then not conducting. On the semiconductor side the phenomena are identical to those in a p-n junction.

The static *V-I* characteristic of a Schottky diode is similar to that of a p-n junction diode (Fig. 2.8) and can be written in the form

$$I_d = I_{D0}(e^{V_d/\varphi_t} - 1), \tag{2.38}$$

where the reverse saturation current is

$$I_{D0} = K_{SB} T^2 e^{-\phi_B/\varphi_t}. \tag{2.39}$$

K_{SB} is a constant which is determined experimentally and ϕ_B is the Schottky barrier. The reverse current of a Schottky diode is three to four orders of magnitude higher than the reverse current of a p-n junction diode of the same surface. For a junction with a surface of S = 100 μm^2 the reverse current is within the limits 2×10^{-14} A and 1 nA, depending on the material used.

Except for the difference in the reverse currents the threshold voltages of a Schottky and a p-n junction diodes are quite different. For a Schottky diode this voltage is typically 0.3 V whereas for a *Si* p-n diode it is approximately 0.6 V. Due to the smaller potential barrier the temperature coefficient of the forward voltage for a Schottky diode is smaller than that of a p-n junction diode (Fig. 2.8b). By neglecting one in the brackets of Eq. (2.38) and having in mind (2.39) it follows

$$\frac{dV_d}{dT}\Big|_{\Delta I_d=0} = \frac{V_d}{T} - \frac{\varphi_t}{T}\left(2 + \frac{\phi_B}{\varphi_t}\right). \tag{2.40}$$

For a Schottky diode with aluminum at room temperature $\phi_B = 0.7$ V. If V_D is taken to be 0.4 V, one obtains that $dV_d/dT = -1.2$ mV/°C. The experimentally obtained characteristics for the p-n junction and Schottky diodes (Fig. 2.8b) show that the temperature coefficient of a Schottky diode is smaller by about 0.4 mV/°C.

Schottky diodes are faster. In p-n junction diodes current is carried predominantly by minority carriers. Owing to their accumulation around the junction a delay arises in the turn-off process. In Schottky diodes the electrons are free charge carriers in metal and majority carriers in semiconductor. Current is thus carried by majority carriers and there is no effect of accumulation of minority carriers. Thanks to this Schottky diodes are considerably faster compared to p-n junction diodes. The recovery time of small-signal Schottky diodes is typically less than 0.1 ns.

2.1.4 The Selection of Pulse Diodes

For a good selection of a diode in each specific circuit, it is necessary to know its operation well, the circuit properties and manufacturer's data. These data are usually given in the form of maximum ratings of the static parameters in the forward and the reverse regions and the parameters of the transient state. Usually the following parameters are given at 25 °C:

- the maximum reverse voltage V_I or V_R (this is the maximum negative voltage still not causing the breakdown),
- the maximum reverse current I_R or I_I (this is the current at V_I),
- the maximum forward dc current I_D or I_F,
- the maximum forward dc voltage V_D at the current I_D,
- the maximum permitted power P_D (often this information is given instead of I_D or V_D),
- the maximum allowed junction temperature $T_{j\max}$ (this is most often the temperature at which the reverse current is not greater than the given value of I_I. Otherwise, the maximum p-n junction temperature is 90 °C for germanium and 175 °C for silicon diodes),
- the diagram of the permitted forward current versus temperature (Fig. 2.9a) and
- the diagram of the permitted power versus case temperature (Fig. 2.9b).

These data are given for diodes regardless of their purpose. In particular, for pulse diodes, the following additional data are given:

- the maximum forward current I_{DM}, when the current through the diode is pulsed (often the pulse width for a given I_{DM} is specified), and
- the maximum recovery time t_{rr} (usually both the forward and the reverse currents of the transient regime are given for which the specified t_{rr} is guaranteed).

Table 2.1 contains the maximum ratings of the basic parameters for some types of power, low-power, and Schottky diodes.

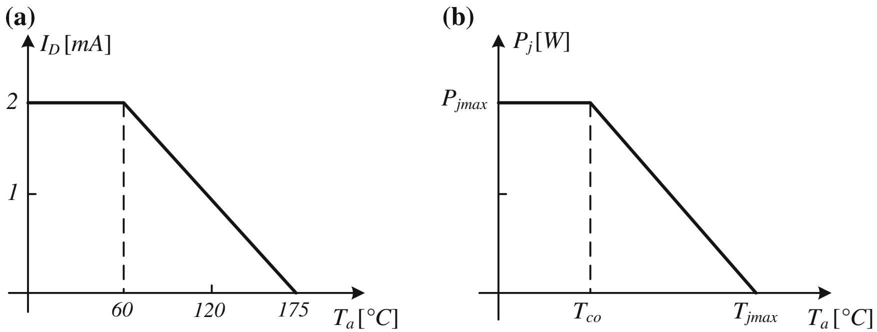

Fig. 2.9 The permitted forward current as a function of the ambient temperature (**a**) and the permitted junction power as a function of the case temperature (**b**)

Table 2.1 The basic parameters of different types of diodes

Type	The characteristics at 25 °C						
	I_p[A]	I_{DM}[A]	V_D[V]	I_D[A]	V_I[V]	I_R/V_I[mA]	$t_{rr,max}$[ns]
	Power diodes						
		10 ms	T_j = 100 °C			T_j = 100 °C	$I_D = I_I$ = 10 mA
BYW 77-50	25	500	0.85	20	50	2.5	35
BYW 78-200	50	1500	0.85	50	200	5	60
BYW 08-100	80	1500	0.92	80	100	5	60
BYT 03-400	3	60	1.3	3	400	0.5	2
BYT08P-300A	8	100	1.3	8	300	2.5	2.2
	Low power diodes						
		8.3					
1N4149	0.2	0.5	–	–	75	50×10^{-5}	4
1N4151	0.2	0.5	1	0.05	45	50×10^{-5}	2
1N4152	0.2	0.5	0.88	0.02	40	50×10^{-5}	2
1N3070	0.2	0.5	1	0.1	200	100×10^{-5}	5
	Schottky diodes						
Bat 17	0.3	–	0.6	10	4	0.25×10^{-3}	<5 (τ < 100 ps)

The dependence of the forward DC current on the ambient temperature T_a (Fig. 2.9a) shows that for $T_a \leq 60$ °C the forward current is constant $I_D = 2$ A in the given example. Above this temperature, the permitted forward current drops and for $T_a = T_{j\max}$ it is zero.

The power dissipated in a diode is $P_D = V_D I_D$ and it behaves as a source of heat. Consequently, the diode temperature increases. Thus, the process of self-destruction is possible as the influence of the current temperature coefficient which is positive prevails over the voltage temperature coefficient which is negative. Above the housing temperature T_{c0} the maximum permitted power of the junction decreases (Fig. 2.9b). In the range $T_{c0} < T < T_{j\max}$

$$\frac{T_{j\max} - T_{C0}}{P_{j\max}} = \frac{T_{j\max} - T_C}{P_j}, \tag{2.41}$$

where T_c is the housing temperature, P_j is the permitted, and $P_{j\max}$ is the maximum permitted junction power. From (2.41) it follows:

$$P_j = \frac{T_{j\max} - T_c}{R_{\mathrm{jc}}}, \tag{2.42}$$

where

$$R_{\mathrm{jc}} = \frac{T_{j\max} - T_{c0}}{P_{j\max}}. \tag{2.43}$$

is the thermal resistance between the junction and the housing. A part of the heat is exchanged between the housing and the ambient. In relation to this, the thermal resistance housing-ambient is defined as the difference between the junction-housing resistance and the total resistance, i.e.,

$$R_{\mathrm{ca}} = \frac{T_{j\max} - T_a}{P_j} - R_{\mathrm{jc}}. \tag{2.44}$$

The removal of heat is facilitated by mounting the diode on a heat sink. The junction temperature is then

$$T_j = P_j\left(R_{\mathrm{jc}} + R_{\mathrm{ch}} + R_{\mathrm{ha}}\right) + T_a, \tag{2.45}$$

where R_{ch} and R_{ha} are the respective resistances housing-heat sink and heat sink-ambient.

2.2 Bipolar Transistor as a Switch

The applications of diodes as switches are quite limited owing to the fact that a diode is a two-terminal device so the control and the controlled circuits are the same. A transistor is a three-terminal device and the control circuit is separated from the load. In accordance with this, it is a standard switching element. In principle, transistors can be connected to a switching circuit in three different configurations:

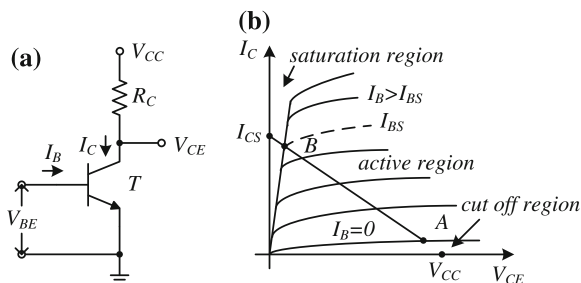

Fig. 2.10 The basic switching circuit (**a**) and operating regions of transistors (**b**)

common-emitter, common-base, or common-collector. As a rule, however, transistors are used as switches in the common-emitter configuration. Namely, in this case it is the highest ratio of the load current (collector current) to the input control current (base current), which maintains the on state of the transistor. In other words, this configuration requires the least power for performing control which is the basic requirement for any switch.

In addition to the transistor being used as the switch, the basic switching circuit comprises a load and a power supply (Fig. 2.10a). Depending upon the position of the operating point the transistor will be in one of the three possible regions: saturation, cut off, or active region (Fig. 2.10b). As a switch the transistor is either in saturation or is cut off. The saturation corresponds to the on-state and cut off to the off-state of the switch. These are the static states of the switch. For the analysis of the parameters of the switch use will be made of the Ebers-Moll equations given in the following form

$$I_C = \alpha_N I_E - I_{\mathrm{C0}}\left(e^{V_{\mathrm{BC}}/(m_c\varphi_t)} - 1\right), \tag{2.46}$$

$$I_E = \alpha_I I_C + I_{\mathrm{E0}}\left(e^{V_{\mathrm{BE}}/(m_c\varphi_t)} - 1\right), \tag{2.47}$$

where α_N and α_I are the respective current gain coefficients of the transistor in the common-base connection for the direct (normal) and reverse modes, I_{C0} is the collector current with the emitter circuit open, I_{E0} is the emitter current with the collector circuit open, m_c and m_e are the respective correction coefficients of the collector and emitter p-n junctions. Like in the case of the diode m_c and m_e are 2 at low currents and 1 at medium currents.

2.2.1 The Cut Off Region

In the cut off region the transistor as a switch is in the off state. It would be ideal if the collector current in this state, i.e., the load current, were equal to zero. In reality,

however, this current does exist. Its value depends on the method the transistors have been switched off. Each of these methods will be analyzed.

1. Both p-n junctions are reverse biased, i.e. $V_{BC} < 0$ and $V_{BE} < 0$

Let $|V_{BC}| \gg m_c\varphi_t$ and $|V_{BE}| \gg m_e\varphi_t$ (these conditions are already fulfilled if V_{BC} and V_{BE} are several 100 mV since φ_t = 26 mV and max$\{m_{e,}\ m_c\}$ = 2). Then:

$$e^{V_{BC}/m_c\varphi_t} \ll 1 \quad \text{and} \quad e^{V_{BE}m_e\varphi_t} \ll 1,$$

and from (2.46) and (2.47) it follows:

$$I_C = \alpha_N I_E + I_{C0} \tag{2.48}$$

$$I_E = \alpha_I I_C - I_{E0}\,. \tag{2.49}$$

From (2.48) and (2.49), having in mind that

$$\alpha_I I_{C0} = \alpha_N I_{E0} \tag{2.50}$$

one obtains that the collector and the emitter currents, when both junctions are reverse biased, are determined by:

$$I_C = \frac{1-\alpha_I}{1-\alpha_I\alpha_N} I_{C0}, \tag{2.51}$$

$$I_E = -\frac{\alpha_I(1-\alpha_N)}{\alpha_N(1-\alpha_N\alpha_I)} I_{C0}. \tag{2.52}$$

Typical values of the current coefficients are α_N = 0.96 − 0.995 and α_I = 0.3 − 0.7. At small currents, these values are several time smaller. Thus, α_N $\alpha_I \ll 1$ and

$$I_c \approx (1-\alpha_I)I_{C0} < I_{C0}\,, \tag{2.53}$$

$$I_E \approx -\frac{\alpha_I}{\beta_N} I_{C0}, \tag{2.54}$$

where

$$\beta_N = \frac{\alpha_N}{1-\alpha_N} \tag{2.55}$$

is the common emitter current gain. The transistor can be replaced by the simplified equivalent circuit (Fig. 2.11a). The negative base current is given by

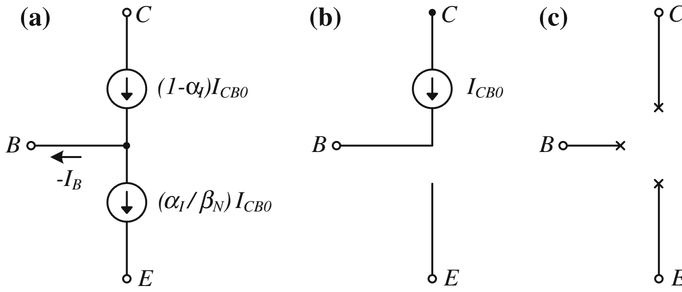

Fig. 2.11 The simplified equivalent circuits of transistor in the cut off region when both junctions are reverse biased

$$I_{\mathrm{B}} \approx -(1-\alpha_I+\alpha_I/\beta_N)I_{\mathrm{C}0}. \tag{2.56}$$

At small currents, β_N ranges from 1 to 5 and $\alpha_I \ll 1$ so that $I_C \approx I_{C0}$, $I_E \approx 0$, and $I_B = -I_{C0}$. The transistor equivalent circuit is shown in Fig. 2.11b. This is, therefore, equivalent to the open emitter. Since $I_{\mathrm{C}} = -I_{\mathrm{B}}$, it is customary that the collector-base current is denoted by I_{CBO} (open emitter collector-base current) and is often called—the reverse base current. It is straightforward to show that $V_{\mathrm{BE}} < 0$ when the emitter is open. Namely, by introducing $I_{\mathrm{E}} = 0$ and $I_{\mathrm{C}} = I_{C0}$ in (2.47) one obtains:

$$V_{\mathrm{BEO}} = m_e\varphi_t \ln(1-\alpha_N) = -m_e\varphi_t \ln(1+\beta_N). \tag{2.57}$$

For $m_e = 2$ and $\beta_N = 3$, $V_{\mathrm{BEO}} = -72$ mV. The current I_{CB0} is temperature dependent and, like the reverse diode current, it doubles with every 10 °C of temperature increase, i.e.

$$I_{\mathrm{CBO}}(T) = I_{\mathrm{CBO}}(T_0)2^{\frac{T-T_0}{10\,^\circ\mathrm{C}}}. \tag{2.58}$$

Usually I_{CB0} at room temperature is of the order of nA and for power transistors of the order of µA. In most practical applications, I_{CBO} can be neglected and the transistor can be considered an open circuit (Fig. 2.11c).

2. The second method of achieving cut off is obtained when: $V_{\mathrm{BE}} = 0$ and $V_{\mathrm{BC}} < 0$

i.e., when the base and emitter are short circuited and the collector junction is reverse biased. For $|V_{\mathrm{BC}}| \gg m_c\varphi_t$ from (2.46) and (2.47) it follows:

$$I_{\mathrm{C}} = \frac{I_{\mathrm{C}0}}{1-\alpha_N\alpha_I}, \tag{2.59}$$

$$I_E = \alpha_I I_C = \frac{\alpha_I}{1-\alpha_N\alpha_I} I_{\mathrm{C}0}. \tag{2.60}$$

Since $\alpha_N\alpha_I \ll 1$, then: $I_C \approx I_{C0}$, $I_E \approx \alpha_I I_{C0}$. The base current is $I_B = I_E - I_C = -(1-\alpha_I)I_{C0}$. Since $\alpha_I \ll 1$, $I_B \approx -I_{C0}$. Therefore, when the base and emitter are short circuited ($V_{BE} = 0$) the currents are approximately as if $V_{BE} < 0$ or the emitter was open, i.e.

$$I_C \approx -I_B = I_{CBO}, I_E \approx 0. \tag{2.61}$$

3. Transistor will be cut off when: $I_B = 0$, $V_{BC} < 0$

i.e., if the base is open and the collector junction is reverse biased. By replacing $I_C = I_E = I_{CEO}$ in (2.46) and since $|V_{BC}| >> m_c\varphi_t$ it follows:

$$I_{CEO} = \frac{I_{C0}}{1-\alpha_N} = (\beta_N + 1)\, I_{C0}\,. \tag{2.62}$$

In this case, the transistor can be replaced by the equivalent circuit of Fig.2.12. Therefore, if the base is open, the collector current is $\beta_N + 1$ times greater than I_{CBO}. It should be emphasized that β_N at small currents is typically from 1 to 5, and $I_{CEO} = (2–6)I_{CBO}$. The open base voltage $V_{BE} = V_{OBE}$ can be obtained from (2.47) by the replacement $I_E = I_C = I_{CEO}$:

$$V_{OBE} = m_e\varphi_t \ln(1 + \beta_N/\beta_I) > 0. \tag{2.63}$$

For instance, for $\beta_N = 3$, $\beta_I = 0.25$ and $m_e = 2$, $V_{OBE} = 133$ mV.

The characteristics of the currents I_C, I_E, I_B versus voltage V_{BE}, for $V_{BC} < 0$ (Fig. 2.13) show that the transistor is cut off when $V_{BE} < V_{OBE}$. In practice it be may be assumed that a transistor is cut off if $V_{BE} < V_{BEt}$, where V_{BEt} is the voltage V_{BE} at the knee of the characteristic $I_B = f(V_{BE})$. V_{BEt} is the conduction threshold voltage and for silicon transistors it is typically 0.5–0.6 V.

Very often it is not convenient to realize the cut off state by $V_{BE} < 0$ or $V_{BE} = 0$. The third case ($I_B = 0$) should be avoided since the current I_{CB0} is relatively large and, as will be shown, the voltage limitations of transistors are then the most significant. For this reason the cut off state is often realized by a resistor R between the base and the emitter (Fig. 2.14a). The resistor R is chosen so that $V_{BE} < V_{OBE}$. Then the base current is negative and it is certain that $V_{BE} > 0$. The smaller V_{BE} the

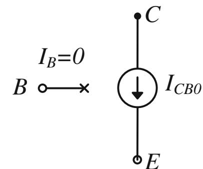

Fig. 2.12 The open base equivalent circuit